RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

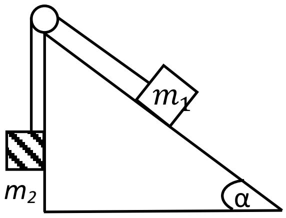
B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2024
SECOND YEAR [BATCH 2022-25]
PHYSICS [Honours]
Paper : CC 9

Date : 22/05/2024
Time : 11 am – 1 pm
Full Marks : 50

Answer **any five** from the following questions: [5×10]

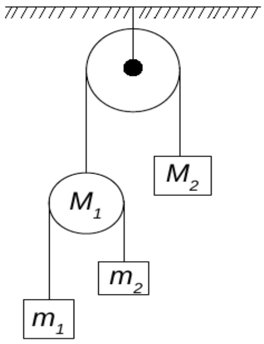
1. a) Prove that the workdone by the constraint forces in case of a rigid body is zero.

   b) Explain the principle of virtual work. Two masses m_1 and m_2 are connected by an inextensible string which passes over a smooth pulley of negligible mass as shown in the figure. Use the principle of virtual work to show that $m_2 = m_1 \sin\alpha$



   c) Give one example and discuss how total virtual workdone due to constraint forces is zero for the following two cases: (i) where constraint force is normal, (ii) where constraint force is not normal to virtual displacement. [3+(1+3)+(1.5+1.5)]

2. Consider the following modified Atwood's machine where two masses M_1 and M_2 are connected by an inextensible string (length l_1) that passes over a pulley. The mass M_1 is itself a pulley from which other two masses m_1 and m_2 are connected over another inextensible string (length l_2).



   a) What are the constraints?

   b) How many generalized coordinates are required to describe the system?

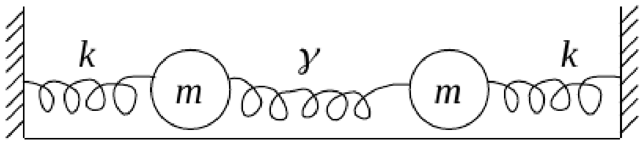
   c) Set up the Lagrangian of the system. Derive the Lagrange's equations of motion. [1+1+(4+4)]

3. a) A particle of mass m is constrained to move on the surface of a cylinder $x^2 + y^2 = r^2$. The particle is subjected to a force directed towards the origin and the magnitude is proportional to the distance of the particle from the origin. Set up the Hamiltonian of the system and show that (i) the angular momentum is conserved. (ii) The particle executes simple harmonic motion along z-axis.

   b) Show that if the Hamiltonian is not explicit function of time, the total energy is conserved.

   c) Using the properties of Poisson brackets, derive the equations of motion of an oscillator moving in the double well potential $U(x) = -\frac{1}{2}kx^2 + \frac{1}{4}\alpha x^4$ where $k > 0$ and $\alpha > 0$. [(2+2+1)+1+4]

4. Consider two oscillators, each of mass m, connected to walls with springs of force constants k. The oscillators are coupled to each other by a spring of force constant γ.



a) Construct the Lagrangian and derive the equations of motion for the two oscillators.

b) Show that the time evolutions of the coordinates of each oscillator can be expressed as superpositions of two simple harmonic motions with frequencies $\omega_1 = \omega_0$ and $\omega_2 = \sqrt{\omega_0^2 + 2\omega_c^2}$, where $\omega_0 = \sqrt{\frac{k}{m}}$ and $\omega_c = \sqrt{\frac{\gamma}{m}}$.

c) From the above solutions, discuss the two special cases (i) symmetric normal mode (in-phase oscillations) and (ii) anti-symmetric normal mode (out-of-phase oscillations). [2+5+(1.5+1.5)]

5. a) Explain why Gallilean transformation is in consistent with the Maxwell's equations. What are the possibilities for these inconsistency? Why the possibility of existence of absolute frame was discarded?

b) A pilot is supposed to fly due East from A to B and then back again to A due to West. The velocity of the plane in air is u' and the velocity of the air with respect to the ground is v. The distance between A and B is l and the plane's air speed u' is constant.

(i) If $v = 0$ (still air), show that the time for the round trip is $t_0 = 2l/u'$.

(ii) Suppose that the air velocity is due East (or West). Show that the time for a round trip is then $t_E = \frac{t_0}{1-v^2/(u')^2}$.

(iii) Suppose that the air velocity is due North (or South). Show that the time for a round trip is then $t_N = \frac{t_0}{\sqrt{1-v^2/(u')^2}}$.

(iv) In part (ii) and (iii) we must assume the $v < u'$. Why?

(v) Draw an analogy to the Michelson-Morley experiment. [(2+2+1)+(1+1+1+1+1)]

6. a) What assumptions other than the relativity principle and the constancy of c, were made in deducing the Lorentz transformation equation? Using all the assumptions derive Lorentz transformation equation.

b) Twin A makes a round trip at 0.6 c to a star 12 light-years away, while twin B stays on the Earth. Each twin sends the other a signal once a year by his own reckoning.

(i) How many signals does A send during the trip? How many does B send?

(ii) How many signals does A receive? How many does B receive? [(1+5)+(2+2)]

7. a) The world line of particle 1 relative to an inertial system is given by $x = ct/2$ and the world line of particle 2 relative to the same system is given by $x = 4ct/5$. (i) Draw the world lines of particles 1 and 2 on a space-time diagram. (ii) What is the velocity of particle 2 relative to particle 1? Draw the world lines of both the particles on a space time diagram of the inertial system of particle 1.

b) Obtain the expression for relativistic kinetic energy of a particle. Why we cannot accelerate a particle to this speed of light?

c) Two particles, each having rest mass m_0, approach each other with speeds 0.8 c and 0.6 c. They collide and stick together. Compute momentum, total energy, total rest mass, kinetic energy after the collision. [(1+2)+(2+1)+(1+1+1+1)]

8. a) A charge q at $x = 0$ accelerates from rest in a uniform electric field E, which is directed along the positive x-axis. Show that the acceleration of the charge is given by $a_x = \frac{qE}{m_0}\left(1 - \frac{u^2}{c^2}\right)^{3/2}$.

b) What potential difference will accelerate electrons to the speed of light, according to classical physics? With this potential difference, what speed would an electron acquire relativistically? What would its mass and kinetic energy be at this speed?

c) Show that the forces transform as

$$F_x' = \frac{F_x - (v/c^2)\vec{u} \cdot \vec{F}}{\left(1 - {u_x v}/{c^2}\right)}, F_y' = \frac{F_y}{\gamma\left(1 - {u_x v}/{c^2}\right)}, F_z' = \frac{F_z}{\gamma\left(1 - {u_x v}/{c^2}\right)}$$

Hint: $m' = m\gamma\left(1 - {u_x v}/{c^2}\right)$, where $\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$. [2+(1+1+2)+4]

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